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I КОЛОКВИЈУМ ИЗ МАТЕМАТИКЕ 2

група 3

1.

$$L=\lim_{(x,y)\to(\infty,\infty)}\frac{x^2y+y^3}{x^4+y^4}\cdot\frac{\ln\left(1+\frac{1}{x^4+y^4}\right)}{\frac{1}{x^4+y^4}}=$$

$t=\frac{1}{x^4+y^4}\to 1$, $t\to 0$

$$=\lim_{(x,y)\to(\infty,\infty)}\frac{x^2y+y^3}{x^4+y^4}\quad\left|\begin{array}{l}x=\rho\cos\varphi\\ y=\rho\sin\varphi\end{array}\right.\Rightarrow \rho\to\infty$$

$$=\lim_{\rho\to\infty}\frac{\rho^3(\cos^2\varphi\sin\varphi+\sin^3\varphi)}{\rho^4(\cos^4\varphi+\sin^4\varphi)}=\lim_{\rho\to\infty}\frac{1}{\rho}=0$$

8п

2. $2z^3+y^3-x^2+xyz-2xz+1=0,\quad z>0,\quad A(1,0)$

$A:\ 2z^3+0-\cancel{1}+0-2z+\cancel{1}=0$

$2z(z^2-1)=0\ \Rightarrow\ z=0\ \vee\ z=1\ \vee\ z=-1$

$z'_x:\ 6z^2z'_x-2x+yz+xyz'_x-2z-2xz'_x=0$

$$z'_x\Big|_A=-\frac{-2x+yz-2z}{6z^2+xy-2x}\Big|_A=-\frac{-2-2}{6-2}=\frac{-(-4)}{4}=1$$

$z'_y:\ 6z^2\cdot z'_y+3y^2+xz+xyz'_y-2xz'_y=0$

$$z'_y\Big|_A=-\frac{3y^2+xz}{6z^2+xy-2x}\Big|_A=-\frac{1}{4}$$

9п

$$z''_{x^2}\Big|_A=-\frac{(-2+y\cdot z'_x-2z'_x)(6z^2+xy-2x)-(-2x+yz-2z)\cdot(12zz'_x+y-2)}{(6z^2+xy-2x)^2}\Big|_A$$

$$=-\frac{(-2-2)\cdot4-(-4)\cdot(12-2)}{4^2}=\frac{4\cdot4-4\cdot10}{16}=\frac{-24}{16}=-\frac{3}{2}$$

$$z''_{y^2}\Big|_A=-\frac{(6y+xz'_y)\cdot(6z^2+xy-2x)-(3y^2+xz)(12zz'_y+x)}{(6z^2+xy-2x)^2}\Big|_A=$$

$$=-\frac{-\frac14\cdot4-1\cdot(-12\cdot\frac14+1)}{4^2}=\frac{+1+(-3+1)}{16}=-\frac{1}{16}$$

$$z''_{xy}\Big|_A=-\frac{(z+yz'_y-2z'_y)\cdot(6z^2+xy-2x)-(-2x+yz-2z)(12zz'_y+x)}{(6z^2+xy-2x)^2}\Big|_A$$

$$=-\frac{(1+2\cdot\frac14)\cdot4-(-4)(-12\cdot\frac14+1)}{4^2}=-\frac{6+4\cdot(-3+1)}{16}=\frac{1}{8}$$

$$T_2=1+(x-1)-\frac14y+\frac12\left(-\frac32(x-1)^2-\frac{1}{16}y^2+2\cdot\frac18(x-1)y\right)\ \to$$

$$T_2 = 1 + (x-1) - \frac{1}{4}y - \frac{3}{4}(x-1)^2 - \frac{1}{32}y^2 + \frac{1}{8}(x-1)y$$

3.) $L = \frac{1}{x^2} + \frac{9}{y^2} + \lambda(xy-27) \Rightarrow x, y \neq 0$

$L'_x = \frac{-2}{x^3} + \lambda y = 0$, $L'_y = -\frac{18}{y^3} + \lambda x = 0$ $\Rightarrow$ $-2 + \lambda x^3 y = 0$, $-18 + \lambda x y^3 = 0$, $xy = 27$ $\Rightarrow$ $x^2 = \frac{2}{\lambda \cdot 27}$, $y^2 = \frac{18}{\lambda \cdot 27}$

$\Rightarrow y^2 = x^2 \cdot 9 \Rightarrow y = \pm 3x$

$xy = x \cdot (\pm 3x) = \pm 3x^2 = 27 \Rightarrow x^2 = \pm 9 \Rightarrow x = \pm 3$

$\Rightarrow y = +3x \rightarrow S_1(3, 9)$, $\lambda_{1,2} = \frac{2}{27 \cdot x^2} = \frac{2}{243}$

$S_2(-3, -9)$

8 $\overline{u}$

$L''_{x^2} = \frac{6}{x^4}$, $L''_{y^2} = \frac{54}{y^4}$, $L''_{xy} = \lambda$

$d^2L(S_{1,2}) = \frac{6}{9}dx^2 + \frac{54}{81}dy^2 + 2 \cdot \frac{2}{243}dx\,dy$

$xy = 27 \,/d \rightarrow y\,dx + x\,dy = 0$

$S_{1,2}: \pm 9dx \pm 3dy = 0 \Rightarrow -3dx = +dy$

$\rightarrow d^2L(S_{1,2}) = \frac{2}{3}dx^2 + \frac{\cancel{54}^{6}}{\cancel{9}}dx^2 - \frac{12}{243}dx^2$

$= \left(\frac{2}{3} + 6 - \frac{4}{81}\right)dx^2 > 0$ min

$$f_{min}(S_{1,2}) = \frac{1}{9} + \frac{9}{81} = \frac{2}{9}$$