|I колоквијум из МАТ2|
|Група 5|

18.04.23.

$\Sigma: 8и$

(1) $L = \lim\limits_{(x,y)\to(0,0)} \left(\underbrace{\left(1+\sin(x^2+y^2)\right)^{\frac{1}{\sin(x^2+y^2)}}}_{\to e} \right)^{\underbrace{\frac{\sin(x^2+y^2)}{x^2+y^2}}_{\to 1} \cdot \frac{3x^2y+5y^4}{\sqrt{x^4+y^6}}}$

$= e^{\overbrace{\lim\limits_{(x,y)\to(0,0)} \frac{3x^2y+5y^4}{\sqrt{x^4+y^6}}}^{\ell}}$

$x = \rho\cos\varphi$
$y = \rho\sin\varphi$ $\quad \rho\to 0$

$\ell = \lim\limits_{\rho\to 0} \frac{3\rho^3\cos^2\varphi\sin\varphi + 5\rho^4\sin\varphi}{\sqrt{\rho^4\cos^4\varphi+\rho^6\sin^6\varphi}} = \lim\limits_{\rho\to 0} \frac{\rho^{\cancel{3}}(3\cos^2\varphi\sin\varphi+5\rho\sin\varphi)}{\cancel{\rho^2}\sqrt{\cos^4\varphi+\rho^2\sin^6\varphi}}$

$= \lim\limits_{\rho\to 0} \left(\underbrace{\rho}_{\to 0} \cdot \frac{3\cos^2\varphi\sin\varphi+5\rho\sin\varphi}{\sqrt{\cos^4\varphi+\rho^2\sin^6\varphi}} \right) \not\to 0 = 0 \Rightarrow \boxed{L = e^0 = 1}$

(2.) $x^2+4y^2-z^2-2xy+3yz+28=0 \quad (*)$

$|'_x: \quad 2x - 2zz'_x - 2y + 3yz'_x = 0 \Rightarrow z'_x = \frac{2x-2y}{2z-3y}$

$|'_y: \quad 8y - 2zz'_y - 2x + 3z + 3yz'_y = 0 \Rightarrow z'_y = \frac{8y-2x+3z}{2z-3y}$

$\begin{cases} 2x-2y=0 \Rightarrow x=y \\ 8y-2x+3z=0 \Rightarrow 6y+3z=0 \Rightarrow z=-2y \end{cases}$

$* \to \quad y^2+4y^2-4y^2-2y^2-6y^2+28=0$

$-7y^2+28=0 \Rightarrow y^2=4 \quad S_1(2,2), z_1=-4; \; S_2(-2,-2), z_2=+4$

$z''_{x^2}\Big|_{S_{1,2}} = \frac{2(2z-3y) - \overbrace{(2x-2y)}^{|_{S_{1,2}} = 0}\cdot(2z'_x)}{(2z-3y)^2}\Big|_{S_{1,2}} =$

$= \frac{2}{2z-3y}\Big|_{S_{1,2}} = \frac{2}{\mp 8 \mp 6} = \begin{cases} -\frac{1}{7} \\ \frac{1}{7} \end{cases}$

$\boxed{\begin{array}{l} S_1 - \max \; (z_1=-4) \\ S_2 - \min \; (z_2=4) \end{array}}$

$z''_{y^2}\Big|_{S_{1,2}} = \frac{(8+3z'_y)(2z-3y) - \overbrace{(8y-2x+3z)}^{|_{S_{1,2}}=0}(2z'_y-3)}{(2z-3y)^2}\Big|_{S_{1,2}} =$

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$= \frac{8+3\overbrace{z'_y}^{|_{S_{1,2}} = 0}}{2z-3y}\Big|_{S_{1,2}} = \frac{8}{\mp 14} = \begin{cases} -\frac{4}{7} \\ \frac{4}{7} \end{cases}$

$z''_{xy}\Big|_{S_{1,2}} = \frac{-2(2z-3y) - \overbrace{(2x-2y)}^{|_{S_{1,2}}=0}(2z'_y-3)}{(2z-3y)^2}\Big|_{S_{1,2}} = \frac{-2}{2z-3y}\Big|_{S_{1,2}} = \begin{cases} \frac{1}{7} \\ -\frac{1}{7} \end{cases}$

$S_1: \quad D_2 = \begin{vmatrix} -\frac{1}{7} & \frac{1}{7} \\ \frac{1}{7} & -\frac{4}{7} \end{vmatrix} = \frac{4}{49} - \frac{1}{49} = \frac{3}{49} > 0$, $\quad D_1 < 0$

$S_2: \quad D_2 = \begin{vmatrix} \frac{1}{7} & -\frac{1}{7} \\ -\frac{1}{7} & \frac{4}{7} \end{vmatrix} = \frac{4}{49} - \frac{1}{49} = \frac{3}{49} > 0$, $\quad D_1 > 0$

3. $L = x^3 + 4y^3 + \lambda(x^2y - 8)$

$L'_x = 3x^2 + 2\lambda xy = 0 \Rightarrow 3x + 2\lambda y = 0$ или $x = 0 \overset{L'_y}{\Rightarrow} y = 0 \overset{\text{e}}{\Rightarrow} 0 = 8 \perp$

$L'_y = 12y^2 + \lambda x^2 = 0$ $\qquad x = -\frac{2}{3}\lambda y$ (1)

$x^2 y = 8$ $\qquad \frac{4}{9}\lambda^2 y^3 = 8 \Rightarrow \lambda^2 y^3 = 18$ (2)

$\Rightarrow 12y^2 + \frac{4}{9}\lambda^3 y^2 = 0$

$y^2\left(12 + \frac{4}{9}\lambda^3\right) = 0$ $\qquad y \neq 0$

$\frac{4}{9}\lambda^3 = -12 \Rightarrow \lambda^3 = -27$

$\lambda = -3$

(2): $9y^3 = 18$

$\Rightarrow y^3 = 2 \Rightarrow y = \sqrt[3]{2}$

(1): $x = -\frac{2}{3}\cdot(-3)\cdot\sqrt[3]{2} = 2\cdot\sqrt[3]{2}$

$S(2\sqrt[3]{2}, \sqrt[3]{2}),\ \lambda = -3$

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$L''_{x^2} = 6x + 2\lambda y$ $\qquad L''_{xy} = 2\lambda x$

$L''_{y^2} = 24y$

$d^2L(S) = (12\sqrt[3]{2} - 6\sqrt[3]{2})dx^2 + 2(-12\sqrt[3]{2})dx\,dy$

$+ 24\sqrt[3]{2}dy^2 = 6\sqrt[3]{2}dx^2 - 24\sqrt[3]{2}dx\,dy + 24\sqrt[3]{2}dy^2$

$= 6\sqrt[3]{2}(dx^2 - 4dx\,dy + 4dy^2) = 6\sqrt[3]{2}(dx - 2dy)^2$

$\neq 0$ $\qquad \not\to 0$

S – min $\qquad f(S) = 16 + 8 = 24$

$x^2 y = 8 \ /d$

$2xy\,dx + x^2 dy = 0$

$4(\sqrt[3]{2})^2 dx + 4(\sqrt[3]{2})^2 dy = 0 \Rightarrow dx = -dy$