МАЗ - 2. КОЛ - 2. ГРУПА   2023.

1] $A=\begin{bmatrix}0&1&-1\\-1&2&-1\\-2&2&-1\end{bmatrix} \Rightarrow A-\lambda E=\begin{bmatrix}-\lambda&1&-1\\-1&2-\lambda&-1\\-2&2&-1-\lambda\end{bmatrix} \Rightarrow \det(A-\lambda E)=\ldots=-(\lambda+1)(\lambda-1)^2=0$

$\Rightarrow \lambda_1=-1,\ \lambda_{2,3}=1$ — СОПСТВ. ВРЕДНОСТИ 1п.

$\lambda_1=-1$: $(A-\lambda_1 E)\cdot M=0$

$$\begin{bmatrix}1&1&-1\\-1&3&-1\\-2&2&0\end{bmatrix}\cdot\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \Rightarrow \left.\begin{matrix}a+b-c=0\\-a+3b-c=0\\-2a+2b=0\end{matrix}\right\} \Rightarrow \ldots \Rightarrow M=\begin{bmatrix}a\\b\\c\end{bmatrix}=a\begin{bmatrix}1\\1\\2\end{bmatrix} \Rightarrow X_1=\begin{bmatrix}1\\1\\2\end{bmatrix}e^{-t}$$ 2п.

$\lambda_{2,3}=1$: $(A-\lambda_{2,3}E)\cdot M_2=0$

$(A-\lambda_{2,3}E)\cdot M_1=M_2$

$$\begin{bmatrix}-1&1&-1\\-1&1&-1\\-2&2&-2\end{bmatrix}\cdot\begin{bmatrix}a_2\\b_2\\c_2\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix},\quad \begin{bmatrix}-1&1&-1\\-1&1&-1\\-2&2&-2\end{bmatrix}\cdot\begin{bmatrix}a_1\\b_1\\c_1\end{bmatrix}=\begin{bmatrix}a_2\\b_2\\c_2\end{bmatrix} \Rightarrow \left.\begin{matrix}-a_2+b_2-c_2=0\\-a_2+b_2-c_2=0\\-2a_2+2b_2-2c_2=0\\-a_1+b_1-c_1=a_2\\-a_1+b_1-c_1=b_2\\-2a_1+2b_1-2c_1=c_2\end{matrix}\right\} \Rightarrow \ldots \Rightarrow M_1=\begin{bmatrix}a_1\\a_1+c_1\\c_1\end{bmatrix},\ M_2=\begin{bmatrix}0\\0\\0\end{bmatrix},\ X_{2,3}=(M_1+M_2t)e^{t}$$

$a_1=1, c_1=0$: $X_2=\begin{bmatrix}1\\1\\0\end{bmatrix}e^{t}$ 2п. $a_1=0, c_1=1$: $X_3=\begin{bmatrix}0\\1\\1\end{bmatrix}e^{t}$ 2п.

$$X=C_1X_1+C_2X_2+C_3X_3 \Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=C_1\begin{bmatrix}1\\1\\2\end{bmatrix}e^{-t}+C_2\begin{bmatrix}1\\1\\0\end{bmatrix}e^{t}+C_3\begin{bmatrix}0\\1\\1\end{bmatrix}e^{t} \Rightarrow \begin{matrix}x=\_\_\_\\y=\_\_\_\\z=\_\_\_\end{matrix}$$ 1п.

3] $\left.\begin{matrix}x'=4x+2y+\sin t\\y'=-3x-y+3\cos t\end{matrix}\right\}\Big/\mathcal{L}$

$\mathcal{L}(x)=X,\ \mathcal{L}(y)=Y \Rightarrow \mathcal{L}(x')=sX-x(0)=sX+1,\ \mathcal{L}(y')=sY-y(0)=sY-1$

$$\left.\begin{matrix}sX+1=4\cdot X+2\cdot Y+\frac{1}{s^2+1}\\sY-1=-3X-Y+\frac{3s}{s^2+1}\end{matrix}\right\}$$ 1п.

$$\left.\begin{matrix}(s-4)\cdot X-2\cdot Y=\frac{-s^2}{s^2+1}\\3\cdot X+(s+1)\cdot Y=\frac{s^2+3s+1}{s^2+1}\end{matrix}\right\} \Rightarrow D=\begin{vmatrix}s-4&-2\\3&s+1\end{vmatrix}=(s-1)(s-2),\ D_X=\begin{vmatrix}\frac{-s^2}{s^2+1}&-2\\\frac{s^2+3s+1}{s^2+1}&s+1\end{vmatrix}=\frac{-s^3+s^2+6s+2}{s^2+1}$$

$$D_Y=\begin{vmatrix}s-4&\frac{-s^2}{s^2+1}\\3&\frac{s^2+3s+1}{s^2+1}\end{vmatrix}=\frac{s^3+2s^2-11s-4}{s^2+1}$$

$$\Rightarrow X=\frac{D_X}{D}=\frac{-s^3+s^2+6s+2}{(s-1)(s-2)(s^2+1)}=\left[\frac{A}{s-1}+\frac{B}{s-2}+\frac{Cs+D}{s^2+1}\right]=\ldots=\frac{-4}{s-1}+\frac{2}{s-2}+\frac{s-2}{s^2+1}$$ 3п

$$\Rightarrow Y=\frac{D_Y}{D}=\frac{s^3+2s^2-11s-4}{(s-1)(s-2)(s^2+1)}=\left[\frac{A}{s-1}+\frac{B}{s-2}+\frac{Cs+D}{s^2+1}\right]=\ldots=\frac{6}{s-1}-\frac{2}{s-2}+\frac{-3s+3}{s^2+1}$$ 3п $\Big/\mathcal{L}^{-1}$

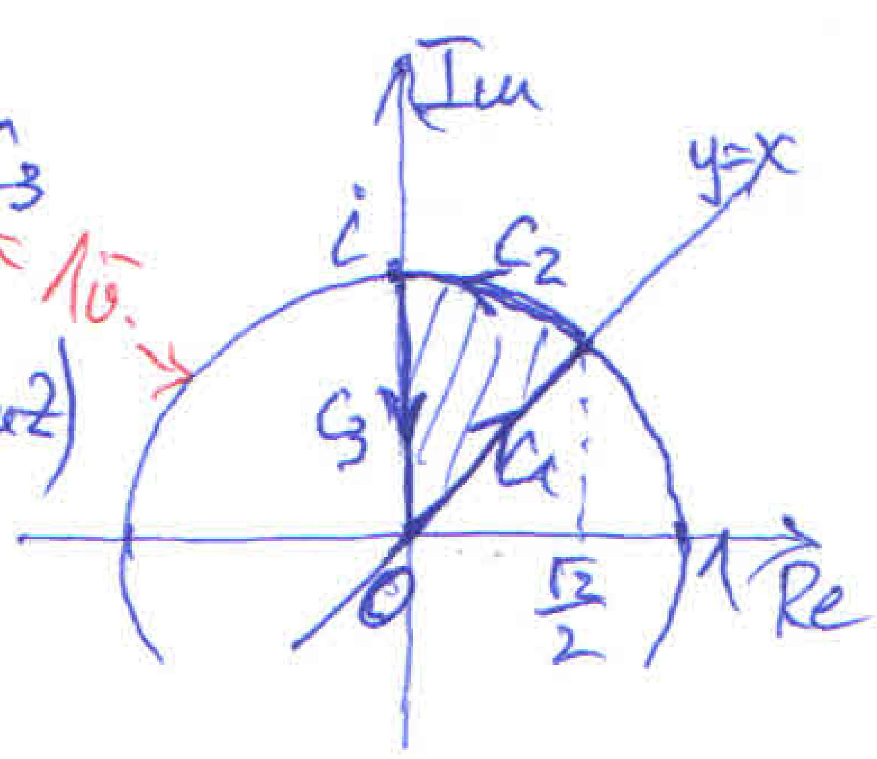
$$\Rightarrow x=\mathcal{L}^{-1}(X)=-4\mathcal{L}^{-1}\left(\frac{1}{s-1}\right)+2\mathcal{L}^{-1}\left(\frac{1}{s-2}\right)+\mathcal{L}^{-1}\left(\frac{s}{s^2+1}\right)-2\mathcal{L}^{-1}\left(\frac{1}{s^2+1}\right)=-4e^{t}+2e^{2t}+\cos t-2\sin t$$

$$\Rightarrow y=\mathcal{L}^{-1}(Y)=6\mathcal{L}^{-1}\left(\frac{1}{s-1}\right)-2\mathcal{L}^{-1}\left(\frac{1}{s-2}\right)-3\mathcal{L}^{-1}\left(\frac{s}{s^2+1}\right)+3\mathcal{L}^{-1}\left(\frac{1}{s^2+1}\right)=6e^{t}-2e^{2t}-3\cos t+3\sin t$$ 1п.

$\boxed{2}$ $C_1: x=y \Rightarrow z=x+ix,\ x\Big|_0^{\frac{\sqrt2}{2}},\ dz=(1+i)dx$

$C=C_1\cup C_2\cup C_3$ 1т.

$f(z)=z\cdot(\operatorname{Re}z+\operatorname{Im}z)$



$$\Rightarrow \int_{C_1} f(z)dz=\int_0^{\sqrt2/2}(1+i)x\cdot(x+x)\cdot(1+i)dx=(1+i)^2\cdot\int_0^{\sqrt2/2}2x^2=$$

$$=2i\cdot\frac{2}{3}x^3\Big|_0^{\sqrt2/2}=\frac{4i}{3}\cdot\left(\frac{2\sqrt2}{8}-0\right)=\underline{\frac{\sqrt2 i}{3}}\ \text{2т.}$$

$C_2: z=e^{it}=\cos t+i\sin t,\ t\Big|_{\pi/4}^{\pi/2},\ dz=ie^{it}dt$

$$\Rightarrow\int_{C_2} f(z)dz=\int_{\pi/4}^{\pi/2}(\cos t+i\sin t)\cdot(\cos t+\sin t)\cdot i(\cos t+i\sin t)dt=i\int_{\pi/4}^{\pi/2}(\cos^2t-\sin^2t+2i\cos t\sin t)\cdot(\cos t+\sin t)dt=$$

$$=i\int_{\pi/4}^{\pi/2}(\cos^3t-\sin^3t+\cos^2t\sin t-\sin^2t\cos t+2i\cos^2t\sin t+2i\sin^2t\cos t)dt=$$

$$=i\int_{\pi/4}^{\pi/2}(1-2\sin^2t+2i\sin^2t)\cos t\,dt+i\int_{\pi/4}^{\pi/2}(-1+2\cos^2t+2i\cos^2t)\sin t\,dt=$$

$$=i\int_{\pi/4}^{\pi/2}\cos t\,dt+i(2i-2)\int_{\pi/4}^{\pi/2}\sin^2t\cos t\,dt-i\int_{\pi/4}^{\pi/2}\sin t\,dt+i(2i+2)\int_{\pi/4}^{\pi/2}\cos^2t\sin t\,dt=\left[z_1=\sin t,\ z_2=\cos t\right]$$

$$=i\sin t\Big|_{\pi/4}^{\pi/2}-(2+2i)\cdot\frac{\sin^3t}{3}\Big|_{\pi/4}^{\pi/2}+i\cos t\Big|_{\pi/4}^{\pi/2}+(2-2i)\cdot\frac{\cos^3t}{3}\Big|_{\pi/4}^{\pi/2}$$

$$=i\left(1-\frac{\sqrt2}{2}\right)-\frac{2+2i}{3}\cdot\left(1-\frac{2\sqrt2}{8}\right)+i\left(0-\frac{\sqrt2}{2}\right)+\frac{2-2i}{3}\cdot\left(0-\frac{2\sqrt2}{8}\right)=\ldots=\underline{\frac{(1-2\sqrt2)i-2}{3}}\ \text{3т.}$$

$\Big[$II Начин: $\displaystyle\int_{C_2}f(z)dz=i\int_{\pi/4}^{\pi/2}e^{2it}\cdot\left(\frac{e^{it}+e^{-it}}{2}+\frac{e^{it}-e^{-it}}{2i}\right)dt=\frac{1+i}{2}\int_{\pi/4}^{\pi/2}e^{3it}dt+\frac{i-1}{2}\int_{\pi/4}^{\pi/2}e^{it}dt=\frac{1+i}{2}\frac{e^{3it}}{3i}\Big|_{\pi/4}^{\pi/2}+\frac{i-1}{2}\frac{e^{it}}{i}\Big|_{\pi/4}^{\pi/2}=$

$$=\frac{1+i}{6i}\cdot\left(e^{\frac{3\pi}{2}i}-e^{\frac{3\pi}{4}i}\right)+\frac{i-1}{2i}\cdot\left(e^{\frac{\pi}{2}i}-e^{\frac{\pi}{4}i}\right)=\frac{1-i}{6}\cdot\left[-i-\left(-\frac{\sqrt2}{2}+\frac{\sqrt2}{2}i\right)\right]+\frac{1+i}{2}\cdot\left[i-\left(\frac{\sqrt2}{2}+\frac{\sqrt2}{2}i\right)\right]=\ldots=\frac{(1-2\sqrt2)i-2}{3}\Big]$$

$C_3: x=0\Rightarrow z=iy,\ y\Big|_1^0,\ dz=i\,dy$

$$\Rightarrow\int_{C_3}f(z)dz=\int_1^0 iy\cdot(0+y)\cdot i\,dy=i^2\int_1^0y^2dy=\int_0^1y^2dy=\frac{y^3}{3}\Big|_0^1=\frac{1-0}{3}=\underline{\frac{1}{3}}\ \text{2т.}$$

$$\Rightarrow\int_{C^+}f(z)dz=\int_{C_1}f(z)dz+\int_{C_2}f(z)dz+\int_{C_3}f(z)dz=\frac{\sqrt2 i}{3}+\frac{(1-2\sqrt2)i-2}{3}+\frac{1}{3}=\ldots=\underline{\frac{(1-\sqrt2)i-1}{3}}\ \text{1т.}$$