МАТЕМАТИКА 3 - I колоквијум

група 2

(24.11.22.)

1. $xy' - y = (x+y)\ln\frac{x+y}{x} \quad /:x$

$$y' = \frac{y}{x} + \left(1+\frac{y}{x}\right)\ln\left(1+\frac{y}{x}\right) \quad (1) \qquad \frac{y}{x} = z \Rightarrow y' = z'x + z$$ (1)

$$z'x + z = z + (1+z)\ln(1+z) \quad (2)$$

$$\frac{dz}{(1+z)\ln(1+z)} = \frac{dx}{x} \qquad \left\lceil \ln(1+z) = t, \quad dt = \frac{dz}{1+z} \right\rfloor$$

$$\frac{dt}{t} = \frac{dx}{x} \Rightarrow \ln|t| = \ln|x| + \ln C$$ (2) (1)

$$\ln(1+z) = xC$$

$$\boxed{\ln\frac{x+y}{x} = xC} \quad (1)$$

$$1+\frac{y}{x} = e^{xC}$$
$$x+y = xe^{xC}$$
$$y = xe^{xC} - x$$

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2. $y''' - 2y'' - y' + 2y = (6x+8)e^{2x}$

$$k^3 - 2k^2 - k + 2 = 0$$
$$k^2(k-2) - (k-2) = (k^2-1)(k-2) = (k-1)(k+1)(k-2) = 0$$

$$y_h = C_1 e^{x} + C_2 e^{-x} + C_3 e^{2x} \quad (2)$$

$$y_p = (Ax+B)\cdot x e^{2x} = (Ax^2 + Bx)e^{2x} \quad (1)$$

$$y_p' = e^{2x}(2Ax^2 + 2Bx + 2Ax + B) = e^{2x}(2Ax^2 + 2(A+B)x + B)$$

$$y_p'' = e^{2x}(4Ax^2 + 4(A+B)x + 2B + 4Ax + 2(A+B)) =$$
$$= e^{2x}(4Ax^2 + (8A+4B)x + 2A + 4B)$$

$$y_p''' = e^{2x}(8Ax^2 + (16A+8B)x + 4A + 8B + 8Ax + 8A + 4B) =$$
$$= e^{2x}(8Ax^2 + (24A+8B)x + 12A + 12B)$$

$$e^{2x}\Big[(8Ax^2+(24A+8B)x+12A+12B)-(8Ax^2+(16A+8B)x+4A+8B)$$
$$-(2Ax^2+(2A+2B)x+B)+(2Ax^2+2Bx)\Big]=(6x+8)e^{2x}$$

$x^2:\ 8A-8A-2A+2A=0$

$x:\ 24A+\cancel{8B}-16A-\cancel{8B}-2A-\cancel{2B}+\cancel{2B}=6$

$6A=6 \Rightarrow A=1$

$8A+3B=8$

$x^0:\ 12A+12B-4A-8B-B=8 \Rightarrow \to B=0$ (2)

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$$\boxed{y_{op}=C_1e^{x}+C_2e^{-x}+e^{2x}(C_3+x^2)}$$ (1)

3. $$\frac{dx}{x^2+1-yt}=\frac{dy}{xt+xy}=\frac{dt}{xt}$$

- $$\frac{dy}{t+y}=\frac{dt}{t} \Rightarrow \frac{dy}{dt}=\frac{t+y}{t} \Rightarrow y'_t=1+\frac{y}{t}$$

$\left(z=\frac{y}{t} \Rightarrow y'=z't+z\right)$ $\quad z't+\cancel{z}=1+\cancel{z} \Rightarrow \frac{dz}{dt}\cdot t=1$

$$dz=\frac{dt}{t}$$

$z=\ln|t|+C_1$

$\underline{y=t\ln|t|+C_1t}$ $\to$ $C_1=\frac{y}{t}-\ln|t|$ (3)

- $$\frac{dx}{x^2+1-t^2\ln|t|-C_1t^2}=\frac{dt}{xt}$$ (1)

$$x'_t=\frac{x^2+1-t^2\ln|t|-C_1t^2}{xt}$$

$$x'-\frac{x}{t}=\frac{1}{x}\left(\frac{1}{t}-t\ln|t|-C_1t\right)\Big/\cdot 2x \quad \left(\begin{array}{l}\check{z}=x^2\\ \check{z}'=2xx'\end{array}\right)$$ (2)

$$\check{z}'-\frac{2\check{z}}{t}=\frac{2}{t}-2t\ln|t|-2C_1t$$

$$\check{z}=e^{\int\frac{2dt}{t}}\left(C_2+\int\left(\frac{2}{t}-2t\ln|t|-2C_1t\right)e^{-2\int\frac{dt}{t}}dt\right)=$$

$$=t^2\left(C_2+\int\left(\frac{2}{t}-2t\ln|t|-2C_1t\right)\frac{dt}{t^2}\right)$$

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$$\check{z} = t^2\left(C_2 + 2\int \frac{dt}{t^3} - 2\int \frac{\ln|t|\,dt}{t} - 2C_1\int \frac{dt}{t}\right)$$

$$= t^2\left(C_2 - \frac{1}{t^2} - \ln^2|t| - 2C_1\ln|t|\right) \quad (2)$$

$$= C_2 t^2 - 1 - t^2\ln^2|t| - 2C_1 t^2\ln|t|$$

$$x^2 = C_2 t^2 - 1 - t^2\ln^2|t| - 2\left(\frac{y}{t} - \ln|t|\right)t^2\ln|t|$$

$$= C_2 t^2 - 1 - t^2\ln^2|t| - 2yt\ln|t| + 2t^2\ln^2|t|$$

(1)

[illegible]

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$$\frac{x\,dx + y\,dy}{(x^3 + x - \cancel{xty}) + \cancel{xty} + xy^2} = \frac{dt}{xt}$$

$$\frac{\frac{1}{2}d(x^2+y^2)}{x^3 + x + xy^2} = \frac{dt}{xt}$$

$$\frac{d(x^2+y^2)}{x^2+1+y^2} = \frac{2\,dt}{t} \qquad (\bar{z} = x^2+y^2)$$

$$\frac{d\bar{z}}{\bar{z}+1} = \frac{2\,dt}{t} \Rightarrow \ln|\bar{z}+1| = 2\ln|t|\cdot C_2$$

$$x^2+y^2+1 = t^2 C_2 \quad (6)$$