М2 4гр. ЈУН 2020.

Број индекса: Име и презиме:

1.	
2.	
3.	
4.	
Σ	

1. $z(\pi,0)=2$ 2п.

$$z'_x=\frac{z\sin x}{2ze^y+y+\cos x} \quad 3п. \qquad z'_x(\pi,0)=0 \quad 1п$$

$$z'_y=\frac{-z^2e^y-z-2y+3}{2ze^y+y+\cos x} \quad 3п. \qquad z'_y(\pi,0)=-1 \quad 1п$$

$$z''_{xx}=\frac{z\cos x+2\sin x\cdot z'_x-2e^y\cdot(z'_x)^2}{2ze^y+y+\cos x} \quad 3п. \qquad z''_{xx}(\pi,0)=-\frac{2}{3} \quad 1п.$$

$$z''_{xy}=\frac{-z'_x-2ze^yz'_x-2z'_xz'_ye^y+z'_y\sin x}{2ze^y+y+\cos x} \quad 3п. \qquad z''_{xy}(\pi,0)=0 \quad 1п$$

$$z''_{yy}=\frac{-4zz'_ye^y-z^2e^y-2z'_y-2-2e^y(z'_y)^2}{2ze^y+y+\cos x} \quad 3п. \qquad z''_{yy}(\pi,0)=\frac{2}{3} \quad 1п.$$

$$T_2(x,y)=z(\pi,0)+\frac{1}{1!}\left(z'_x(\pi,0)(x-\pi)+z'_y(\pi,0)\cdot y\right)+\frac{1}{2!}\left(z''_{xx}(\pi,0)(x-\pi)^2+2z''_{xy}(\pi,0)(x-\pi)y+z''_{yy}(\pi,0)y^2\right)$$

1п.

$$T_2(x,y)=2-y-\frac{1}{3}(x-\pi)^2+\frac{1}{3}y^2 \quad 2п.$$

2. $F(x,y,\lambda)=xy-3+\lambda(x^3+y^3+16)$ 1п.

$$\left.\begin{aligned}F'_x&=y+3\lambda x^2=0\\ F'_y&=x+3\lambda y^2=0\end{aligned}\right\} - \longrightarrow -(x-y)+3\lambda(x^2-y^2)=0$$

$$(x-y)(-1+3\lambda(x+y))=0$$

$F'_\lambda=x^3+y^3+16=0$ 6п

$x-y=0$, $x=y$, $2x^3=-16$, $x^3=-8$, $x=-2=y$, $S(-2,-2)$ 3п., $\lambda=1/6$

$3\lambda=\frac{1}{x+y}$, $x+y\neq0$ ⋮ 2п.

$F''_{xx}=6\lambda x$

$F''_{xy}=1$

$F''_{yy}=6\lambda y$ 3п.

$d^2F(S)=-2dx^2+2dxdy-2dy^2$ 2п.

$\varphi=x^3+y^3+16=0$

$d\varphi=3x^2dx+3y^2dy=0$

$d\varphi(S)=12dx+12dy=0$

$dy=-dx$ 3п.

$d^2F(S)=-6dx^2<0$, $dx\neq0$ 2п. + 1п.

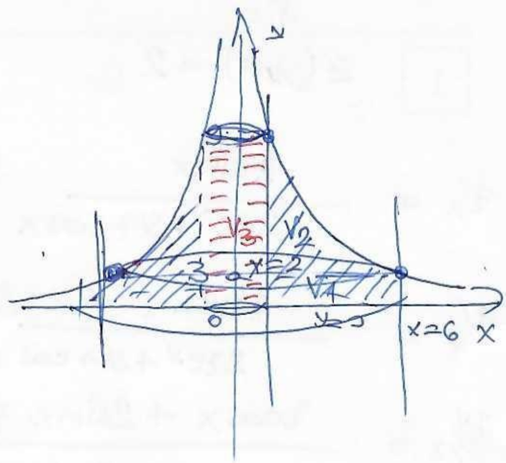
$\downarrow$

max!

$f_{max}=f(-2,-2)=1$

3.

$$V_F = V_1 + V_2 - V_3 \quad 2n.$$



$$V_1 = \pi \int_0^3 6^2 dy = 36\pi \, y \Big|_0^3 = 108\pi \quad 7n.$$

$$V_2 = \pi \int_3^9 \frac{18^2}{y^2} dy = \pi \cdot -\frac{324}{y} \Big|_3^9 = 72\pi \quad 7n.$$

$$V_3 = \pi \int_0^9 2^2 dy = 4\pi \, y \Big|_0^9 = 36\pi \quad 7n.$$

$$V_F = 108\pi + 72\pi - 36\pi$$

$$V_F = 144\pi \quad 2n.$$

4.

$$\iint_D \frac{1}{y} dx dy = \int_{-4}^{4} dx \int_{x^2+4}^{36-x^2} \frac{1}{y} dy = \int_{-4}^{4} \left(\ln y \Big|_{x^2+4}^{36-x^2} \right) dx \quad 2n.$$

$D: -4 \le x \le 4$

$x^2+4 \le y \le 36 - x^2$

$$= \int_{-4}^{4} \ln \frac{36-x^2}{x^2+4} dx = 2 \int_0^4 \ln \frac{36-x^2}{x^2+4} dx \quad 4n.$$

$u = \ln \frac{36-x^2}{x^2+4}$

$du = \frac{-80x}{(x^2+4)(36-x^2)} dx$

$dv = dx$

$v = x$ 4n.

$$= 2x \ln \frac{36-x^2}{x^2+4} \Big|_0^4 - 2 \int_0^4 \frac{-80x^2}{(x^2+4)(36-x^2)} dx \quad 2n.$$

$\ln 1 = 0$

$$= \cancel{2 \cdot 4 \cdot \ln \frac{20}{20}} - \cancel{0 \cdot \ln \frac{36}{4}} + 2 \int_0^4 \frac{80x^2}{(6-x)(x+6)(x^2+4)} dx$$

$$= \int_0^4 \frac{12dx}{6-x} + \int_0^4 \frac{12dx}{x+6} + \int_0^4 \frac{-16dx}{x^2+4}$$

$$\frac{80x^2}{(6-x)(6+x)(x^2+4)} = \frac{A}{6-x} + \frac{B}{x+6} + \frac{Cx+D}{x^2+4}$$

$$\left.\begin{array}{l} A - B - C = 0 \\ 6A + 6B - D = 80 \\ 4A - 4B + 36C = 0 \\ 24A + 24B + 36D = 0 \end{array}\right\} \quad \begin{array}{l} A = B = 6 \\ C = 0 \\ D = -8 \end{array} \quad 6n.$$

$$= -12 \ln(6-x) \Big|_0^4 + 12 \ln(x+6) \Big|_0^4$$

$$- \frac{16}{2} \operatorname{arctg} \frac{x}{2} \Big|_0^4$$

$$= 12 \ln \frac{x+6}{6-x} \Big|_0^4 - 8 \operatorname{arctg} \frac{x}{2} \Big|_0^4$$

$$= 12 \ln 5 - 8 \operatorname{arctg} 2 \quad 7n.$$