Математика 2

3. група

(Јун 2020.)

Број индекса: Име и презиме:

1	25
2	25
3	25
~~4~~	25
Σ	100

1. $z^2+x^2-6xy+xz+18y^2-2=0$

$$z'_x=\frac{6y-2x-z}{2z+x}=0$$

$$z'_y=\frac{-36y+6x}{2z+x}=0$$

(6)

Из система једначина

$$\left.\begin{aligned} &6y-2x-z=0\\ &x-6y=0\\ &z^2+x^2-6xy+xz+18y^2-2=0\end{aligned}\right\}$$

добијамо стационарне тачке $S_1(2,\frac{1}{3})$ и $S_2(-2,-\frac{1}{3})$

$f(S_1)=-2$ $\quad f(S_2)=2$

(6)

$$z''_{x^2}=\frac{-2-z'_x-z'_x(2z'_x+1)}{2z+x}$$

$$z''_{xy}=\frac{6-z'_y-2z'_xz'_y}{2z+x}$$

$$z''_{y^2}=\frac{-36-2(z'_y)^2}{2z+x}$$

Како је $z'_x(S_1)=z'_x(S_2)=0$

$z'_y(S_1)=z'_y(S_2)=0$

добијамо да је

$A=z''_{x^2}(S_1)=1$ | $A=z''_{x^2}(S_2)=-1$

$B=z''_{xy}(S_1)=-3$ | $B=z''_{xy}(S_2)=3$

$C=z''_{y^2}(S_1)=18$ | $C=z''_{y^2}(S_2)=-18$

(9)

S_1: $A>0$

$AC-B^2=9>0$

$\Rightarrow f_{min}=f(S_1)=-2$

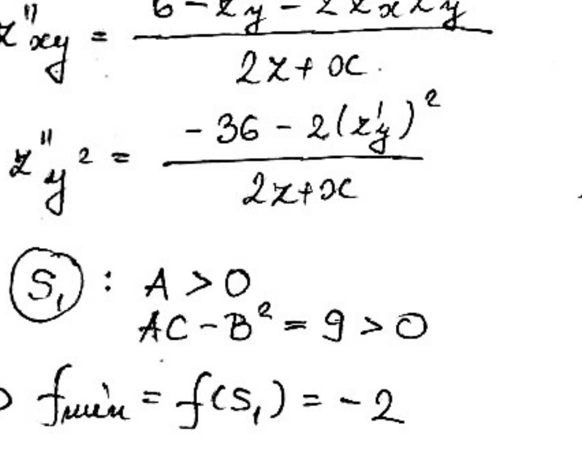
S_2: $A<0$

$AC-B^2=9>0$

$\Rightarrow f_{max}=f(S_2)=2$

(4)

2. $f(x,y)=x^2+y^2-2x$ $\quad D=\{(x,y): x\geq y^2;\ y\leq 2-x\}$



1° стационарне тачке на $\mathring{D}$ (унутрашњ.)

$f'_x=2x-2=0$

$f'_y=2y=0$

$\Rightarrow S_1(1,0)$ (4)

2° граница $x=y^2$

$\varphi(y)=f(y^2,y)=y^4-y^2$

$\varphi'(y)=4y^3-2y=0$

$\to S_2(0,0)$, $S_3(\frac{1}{2},\frac{1}{\sqrt2})$, $S_4(\frac{1}{2},-\frac{1}{\sqrt2})$ (5)

3° граница $y = 2-x$

$\psi(x) = f(x, 2-x) = 2x^2-6x+4$

$\psi'(x) = 4x-6 \Rightarrow S_5\left(\frac{3}{2}, \frac{1}{2}\right)$ (5)

4° темена

$\left.\begin{array}{l} x=y^2 \\ x+y=2 \end{array}\right\}$ решаваме система добиваме $A(1,1)$ и $B(4,-2)$ (5)

$f(A) = 0$

$f(B) = 12$

$f(S_1) = -1$

$f(S_2) = 0$

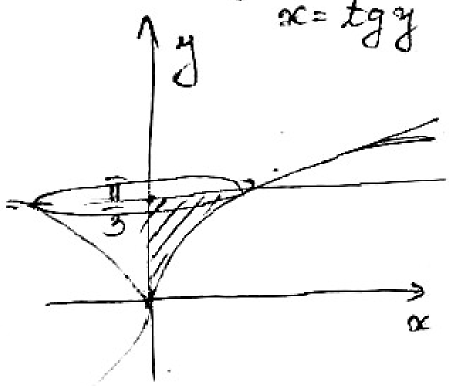
$f(S_3) = f(S_4) = -\frac{1}{4}$

$f(S_5) = -\frac{1}{2}$

$\Rightarrow \min\limits_D f = f(S_1) = -1$, $\max\limits_D f = f(B) = 12$ (4)

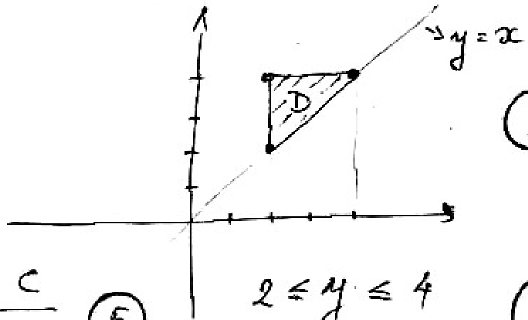
$\boxed{3.}$ $y = \operatorname{arctg} x,\ x=0,\ y=\frac{\pi}{3}$ } око Oy оске (25)

$x = \operatorname{tg} y$



$$V_y = \pi \int_0^{\pi/3} \operatorname{tg}^2 y \, dy$$
$$= \pi \int_0^{\pi/3} \frac{\sin^2 y}{\cos^2 y} dy = \pi \int_0^{\pi/3} \frac{1-\cos^2 y}{\cos^2 y} dy$$
$$= \pi\left(\int_0^{\pi/3} \frac{1}{\cos^2 y} dy - \int_0^{\pi/3} dy\right) = \pi\left(\operatorname{tg} y - y\right)\Big|_0^{\pi/3}$$
$$= \pi\left(\operatorname{tg}\frac{\pi}{3} - \frac{\pi}{3} - \operatorname{tg} 0 - 0\right) = \pi\left(\sqrt{3} - \frac{\pi}{3}\right)$$

$\boxed{4.}$ $\iint\limits_D \frac{1}{y^3+2y^2} dx\,dy$ (2)



$2 \le y \le 4$

$2 \le x \le y$ (5)

$$= \int_2^4 dy \int_2^y \frac{1}{y^3+2y^2} dx = \int_2^4 \left(\frac{1}{y^3+2y^2} \cdot x \Big|_2^y\right) dy$$
$$= \int_2^4 \frac{y-2}{y^2(y+2)} dy = \left[\frac{y-2}{y^2(y+2)} = \frac{A}{y} + \frac{B}{y^2} + \frac{C}{y+2} \Rightarrow A=1,\ B=-1,\ C=-1\right]$$ (5)
$$= \int_2^4 \left(\frac{1}{y} - \frac{1}{y^2} - \frac{1}{y+2}\right) dy$$
$$= \left(\ln y + \frac{1}{y} - \ln(y+2)\right)\Big|_2^4$$
$$= \ln 4 + \frac{1}{4} - \ln 6 - \ln 2 - \frac{1}{2} + \ln 4$$
$$= 2\ln 2 - \ln 3 - \frac{1}{4}$$
$$= \ln\frac{4}{3} - \frac{1}{4}$$ (13)