MD 2гр. ЈУН 2020.

Број индекса: Име и презиме:

$A(0,\frac{\pi}{2})$

1.	
2.	
3.	
4.	
Σ	

$\boxed{1.}$ $z^2e^x + xz - z\sin y + x^2 - 3x = 2,\ z<0$

$z(0,\pi/2) = -1$ 2п.

$z^2 - z - 2 = 0$

$\cancel{z_1 = 2}$ $z_2 = -1$

$$z'_x = \frac{3-2x-z-z^2e^x}{2ze^x - \sin y + x} \quad 3п. \qquad z'_x(A) = -1 \quad 1п.$$

$$z'_y = \frac{z\cos y}{2ze^x - \sin y + x} \quad 3п. \qquad z'_y(A) = 0 \quad 1п.$$

$$z''_{xx} = \frac{-2-2z'_x - 4zz'_xe^x - z^2e^x - 2(z'_x)^2e^x}{2ze^x - \sin y + x} \quad 3п. \qquad z''_{xx}(A) = \frac{7}{3} \quad 1п.$$

$$z''_{xy} = \frac{-z'_y - 2zz'_ye^x + z'_x\cos y + 2z'_xz'_ye^x}{2ze^x - \sin y + x} \quad 3п. \qquad z''_{xy}(A) = 0 \quad 1п.$$

$$z''_{yy} = \frac{2z'_y\cos y - z\sin x - 2(z'_y)^2e^x}{2ze^x - \sin y + x} \quad 3п. \qquad z''_{yy}(A) = -1/3 \quad 1п.$$

$$T_2(x,y) = -1 - x + \frac{1}{2}\left(\frac{7}{3}x^2 - \frac{1}{3}\left(y-\frac{\pi}{2}\right)^2\right) = -1 - x + \frac{7}{6}x^2 - \frac{1}{6}\left(y-\frac{\pi}{2}\right)^2$$

1+2п.

$\boxed{2.}$ $F(x,y,\lambda) = xy + 3 + \lambda(x^3 - y^3 + 16)$ 1п.

$F'_x = y + \lambda\cdot 3x^2 = 0$

$F'_y = x - \lambda\cdot 3y^2 = 0$

$F'_\lambda = x^3 - y^3 + 16 = 0$ 6п.

$+ \to (x+y)(1+3\lambda(x-y)) = 0$

$x+y=0 \quad \vee \quad 1+3\lambda(x-y)=0$

$\vdots$ 2п.

$\emptyset$

$y = -x$

$2x^3 = -16$

$x^3 = -8,\ x = -2$ 3п.

$S(-2,2)\quad \lambda = -\frac{1}{6}$

$F''_{xx} = 6\lambda x$

$F''_{xy} = 1$ 3п.

$F''_{yy} = -6\lambda y$

$d^2F = 6\lambda x\,dx^2 + 2dx\,dy - 6\lambda y\,dy^2$

$d^2F(S) = 2dx^2 + 2dx\,dy + 2dy^2$ 2п. $\to d^2F(S) = 6dx^2 > 0$ 2п.

$\downarrow$

min

$\varphi = x^3 - y^3 + 16$

$d\varphi = 3x^2dx - 3y^2dy = 0$

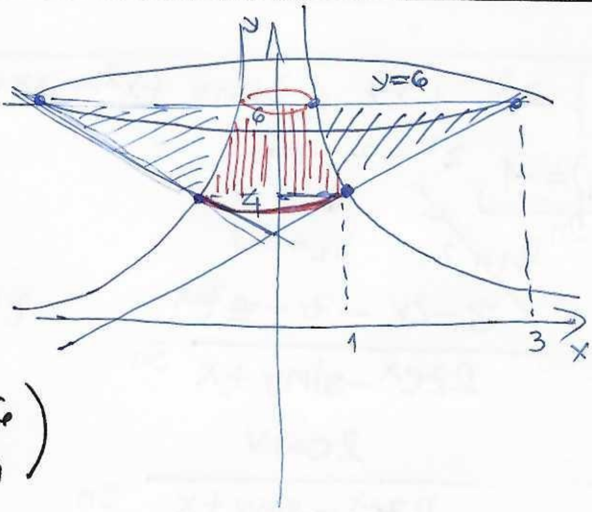
$12dx - 12dy = 0$

$dy = dx$ 2п.

$f_{min} = f(-2,2) = -1$ 1п.

3.

$$V_T = V_1 - V_2 \quad 3п.$$



$$V_1 = \pi \int_4^6 (3-y)^2 dy$$

$$= \pi \int_4^6 (9-6y+y^2) dy$$

$$= \pi \left(9y \Big|_4^6 - 3y^2 \Big|_4^6 + \frac{y^3}{3} \Big|_4^6 \right)$$

$$= \pi \cdot \frac{26}{3} \quad 10п.$$

$$V_2 = \pi \int_4^6 \left(\frac{4}{y}\right)^2 dy = \pi \cdot \int_4^6 \frac{16}{y^2} dy = \pi \cdot 16 \int y^{-2} dy$$

$$= 16\pi \frac{y^{-1}}{-1} \Big|_4^6 = 16\pi \cdot \frac{1}{12} = \frac{4\pi}{3} \quad 10п.$$

$$V_T = \frac{26}{3}\pi - \frac{4}{3}\pi = \frac{22\pi}{3} \quad 2п.$$

4.

$$\iint_D \frac{1}{y} dx dy = \int_{-2}^{2} dx \int_{x^2+1}^{9-x^2} \frac{1}{y} dy \; 2п. = \int_{-2}^{2} dx \left(\ln y \Big|_{x^2-1}^{9-x^2} \right)$$

$D: x^2+1 \leq y \leq 9-x^2$, $-2 \leq x \leq 2$

$$= \int_{-2}^{2} \ln \frac{9-x^2}{x^2+1} dx = 2 \cdot \int_0^2 \ln \frac{9-x^2}{x^2+1} dx \quad 4п.$$

$$u = \ln \frac{9-x^2}{x^2+1}$$

$$du = \frac{-20x}{(x^2+1)(9-x^2)} dx$$

$$dv = dx$$

$$v = x \quad 4п.$$

$$= 2 \left(x \ln \frac{9-x^2}{x^2+1} \Big|_0^2 - \int_0^2 \frac{-20x^2}{(x^2+1)(9-x^2)} dx \right) \quad 2п.$$

$$= 2 \cdot 2 \cdot \ln \frac{9-4}{4+1} - 2 \cdot 0 \cdot \ln \frac{9}{1} + 2 \int_0^2 \frac{20x^2 \, dx}{(x^2+1)(9-x^2)}$$

$$= 4 \cancel{\ln 1}_{0} + 2 \left(\int_0^2 \frac{3dx}{3-x} + \int_0^2 \frac{3dx}{x+3} + \int_0^2 \frac{-2dx}{x^2+1} \right)$$

$$= -2 \cdot 3 \ln(3-x) \Big|_0^2 + 2 \cdot 3 \ln(x+3) \Big|_0^2 - 4 \operatorname{arctg} x \Big|_0^2$$

$$\frac{20x^2}{(x^2+1)(9-x^2)} = \frac{A}{3-x} + \frac{B}{x+3} + \frac{Cx+D}{x^2+1}$$

$$= -6 \ln \frac{3-x}{3+x} \Big|_0^2 - 4 \operatorname{arctg} x \Big|_0^2$$

$$\left.\begin{array}{l} A-B-C=0 \\ 3A+3B-D=20 \\ A-B+9C=0 \\ 3A+3B+9D=0 \end{array}\right\} \to \boxed{\begin{array}{l} A=B=3 \\ C=0 \\ D=-2 \end{array}} \quad 6п.$$

$$= -6 \ln \frac{1}{5} - \cancel{6 \ln \frac{3}{3}} - 4 \operatorname{arctg} 2$$

$$= 6 \ln 5 - 4 \operatorname{arctg} 2 \quad 7п.$$